Computational modeling of concrete flow: General overview

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COMPUTATIONAL MODELING OF CONCRETE FLOW: STATE OF THE ART

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1. INTRODUCTION

To benefit from the full potential of fluid concretes such as Self-Compacting Concrete (SCC) tools for prediction of the form filling of SCC are needed. Such tools should take into account the properties of the concrete, the shape and size of the structural element, the position of rebars, and the casting technique. The present lack of such tools may lead to selection of highly flowable mixes with tendency to segregate or mixes without high static and dynamic stability, increasing the risk of improper form filling. Although substantial progress has been made in the field of fluid concretes, we must not forget that the most suitable concrete to cast a given element is a concrete which is just sufficiently fluid to fill the formwork. Additional and thus unnecessary fluidity will always have a cost, e.g. in terms of increased super-plasticizer amount, increased porosity causing loss of mechanical resistance and durability and increased risk of segregation.

Important requirements to the hardened concrete are total form filling and bond to reinforcement, homogeneity with regard to paste composition, aggregate distribution and air void content, and high quality surfaces without surface air voids ("blowholes"). All these aspects condition the future hardened properties of the material. Segregation could increase the local porosity and thus the permeability of the concrete to aggressive substances. Varying content of cement paste causes heterogeneous shrinkage and creep in a given concrete element. Moreover, high heterogeneity will increase the probability that these time-dependent phenomenon yield high internal stress gradients and thus cracking.

Computational modeling of flow could be used for simulation of e.g. total form filling and detailed flow behavior as particle migration and formation of granular arches between reinforcement ("blocking"). But computational modeling of flow could also be a potential tool for understanding the rheological behaviour of concrete and a tool for mix proportioning. Progresses in the correlation between mix proportioning and rheological parameters would of course result but, moreover, the entire approach to mix proportioning could be improved. Indeed, just as numerical simulations of the loading of concrete structures allow a civil engineer to identify a minimum needed mechanical strength, numerical simulation of the casting process could allow the same engineer to specify a minimum workability of the fresh concrete that could ensure the proper filling of a given formwork.

This paper describes the present status regarding computational modeling of the flow of fresh concrete. Fresh concrete is a suspension of particles in a matrix. Depending on the purpose of the simulation and behavior of the concrete the scales at which the solid components of the concrete

(cement and other binders, fine aggregates, coarse aggregates) are considered as particles or belonging to the matrix vary. The techniques that can be found in the literature may be divided into three main families: single fluid simulations, numerical modeling of discrete particle flow, and numerical techniques allowing the modeling of particles suspended in a fluid. In the following, the general concept of identified methods will be described. Pros and cons for each technique will be given along with examples and references to applications to fresh cementitious materials.

2. PHYSICAL PHENOMENA TO BE MODELED

The problem to be dealt with is not trivial. Concrete casting (and testing of e.g. slump flow or flow in L-Box) can be described as a free surface flow of a non-Newtonian liquid/granular solid. The flow is by nature transient. Moreover, concrete is constituted of particles from the nanoscale to the centimetric scale and its behavior is developing either reversibly (thixotropy) or unreversibly (hydration process). Most of the time, it is assumed that fresh concrete behaves as a yield stress fluid. The Bingham or Herschell Bulkley models are the most common models, but in order to choose this type of modeling, it is necessary to assume that concrete can be considered a homogeneous single fluid. Identifying concrete as a single fluid means that, in any two parts of the observed volume, we should find similar ensemble of **components** [1]. If a physical quantity q (*for instance: velocity*) is studied, the minimum scale on which one can reasonably observe the system and effectively consider it as a single fluid thus corresponds to the point beyond which the average of q no longer varies when this scale further increases (see Fig. 1). The volume corresponding to this scale of observation will be an **elementary part** of the material (as opposed to a **component**). However, it is worth noting that, when further increasing the scale of observation, macroscopic variations start to play a role (see Fig. 1). These variations are precisely what we want to study. The appropriate scale of observation is thus situated between the range of rapid variations of q due to matter discontinuity (*for instance: the variations in velocity between the aggregates and the cement paste*) and the range of macroscopic variations (*for instance the velocity field gradient*). When these two ranges coincide, it is not possible to consider the system as a single fluid under the continuum assumption. In this particular situation, the flow is said to be in the "discrete" regime. The scale of observation is thus of great importance to choose whether or not a single fluid approach is licit.. The order of magnitude of a formwork smallest characteristic size is around 0.1m while the order of magnitude of the size of the coarsest particles is around 0.01m. This means that, if, as a first approximation, the presence of the rebars is neglected, the flow in a typical formwork can be considered as the flow of a single fluid and a discrete Modeling approach is thus not needed.

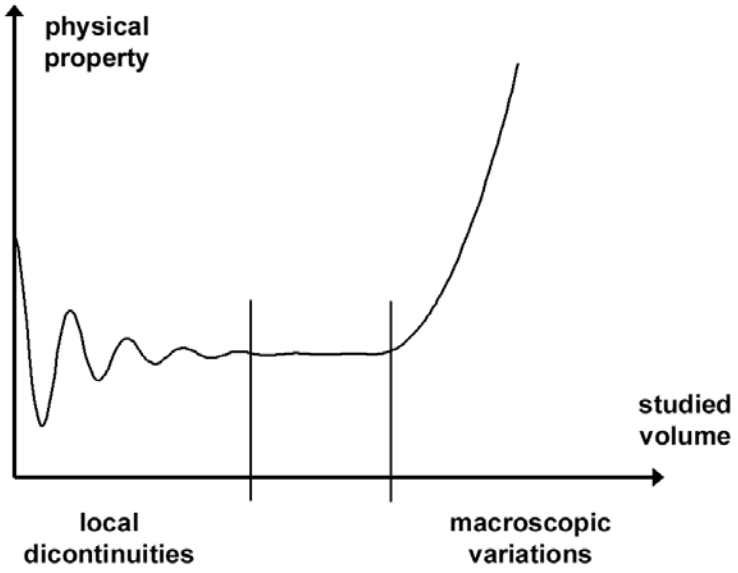

Figure 1. Variation of a physical property averaged over a given volume of material as a function of the volume size.

However, several reasons can lead researchers and engineers to the use of discrete methods or other alternative tools:
_ if the objective is to understand the correlation between rheology and mix composition.
_ if it seems necessary to take into account the presence of the steel bars as, in this case, the studied scale is of the same order as the size of the constitutive grains.
_ if the prediction of the segregation induced by the flow is needed. Indeed, the single fluid techniques rely on the assumption that, of course, the medium is homogenous but, moreover, that it stays homogeneous during flow. When segregation of the coarsest particles is the object of the study, it is thus necessary to introduce particles.

3. SINGLE FLUID SIMULATIONS

Single fluid simulations have mostly been used to model the flow of fresh concrete during testing. However, a few examples of computational modeling of full-scale castings assuming single fluid behavior can also be found in recent papers. The two main numerical difficulties in connection with single fluid simulations are the yield stress behavior of the material and the free surface displacement. Indeed, it is the apparent viscosity of the material that is, most of time, applied in the Navier Stokes equations in order to obtain a numerical solution of the problem. However, the apparent viscosity of a yield stress fluid approaches infinity when the shear rate (or more generally in 3D the strain rate) approaches zero. It is necessary to avoid this indetermination of the deformation state below the yield stress in zones where flow stops or starts, which are most of the time the zones of interest. Moreover, as any continuum mechanics methods, single fluid simulation requires a clear definition of the boundary conditions. Fresh concrete displaying a moving free boundary is thus particularly delicate to simulate.

3.1. Simulations of the testing of fresh concrete

Mori and Tanigawa [2] used the so-called Viscoplastic Finite Element Method (VFEM) and the Viscoplastic Divided Element Method (VDEM) to simulate flow of fresh concrete. Both VFEM and VDEM assume that concrete can be described as a homogeneous single fluid with given rheological properties. In VFEM the fresh concrete is divided into elements in which the deformation is calculated, and the flow is described by displacement of nodal points. In VDEM, space is divided into elements and cells, which are either empty or full, and the flow is described by the displacement of virtual markers. However, the fixed position of nodal points allows reinforcement and complicated boundary conditions to be simulated. Both methods were found applicable for simulation of various test methods and at reasonable computation time.
VFEM was used by Kurokawa et al. [3] to evaluate factors affecting the slump flow of fresh concrete. They used the Bingham model and fitted the rheological properties to experimental results.
In [4] and [5], simulations of SCC flow during L-box and slump flow test were also based on a single fluid approach assuming Bingham behavior. The simulation approach applied was based on the Galerkin FEM formulation of the Navier-Stokes' equation and included moving boundaries, and was undertaken by the code Fidap. Combined simulations and experiments indicated that the slump flow test and the flow in an L-box with reinforcement can be simulated assuming an ideal Bingham behavior and rheological properties measured in a BML rheometer. It is, however, necessary to include boundary conditions such as the speed of lifting the cone (or gate) and to use a 3D model for simulating the flow in the L-box.

Wallevik in [6] has given a detailed description of numerical simulation of the flow in selected rheometers. The simulations are based on the assumption of a viscoplastic material, flowing either under steady state or time dependent ("transient") conditions. Wallevik used a combination of several different techniques to describe the viscoplastic behavior of the concrete. Using his own and freely available numerical software, he simulated velocity and shear stress profiles for various viscometer configurations. The computational modeling was used for the comparison of rheometers with regard to, among others, variations in shear rate and particle migration.
Roussel [7, 8] used the computational fluid mechanics code Flow 3D® to perform 3D simulations of different slump test methods. An elasto-viscoplastic model was used to describe the fluid behavior of concrete with yield stresses between 25 and 5500 Pa, assuming an incompressible and elastic solid up to the yield stress and a Bingham fluid beyond that as well as no sliding at the base. Good agreement between numerical and experimental results was obtained for the mini cone test and for the ASTM tests. It has to be noted that in order to obtain this good quantitative agreement, the author like Thrane *et al.* in [4] and [5] had to implement a proper three dimensional yield criterion. A 3D Bingham model was thus used to describe the tested fluid behaviour. The obtained numerical results were compared with two analytical solutions valid in two asymptotic cases: when there is a very small slump (purely extensional flow) and when there is a large spread (purely shearing flow). Examples of two dimensional predicted shapes are shown in Fig. 2 for the ASTM Abrams cone. The presence of an unyielded zone (usual in this type of simulation) can be noted. The calculated values of the slump confirmed the fact that slump (i.e. final shape) only depends on yield stress and density.

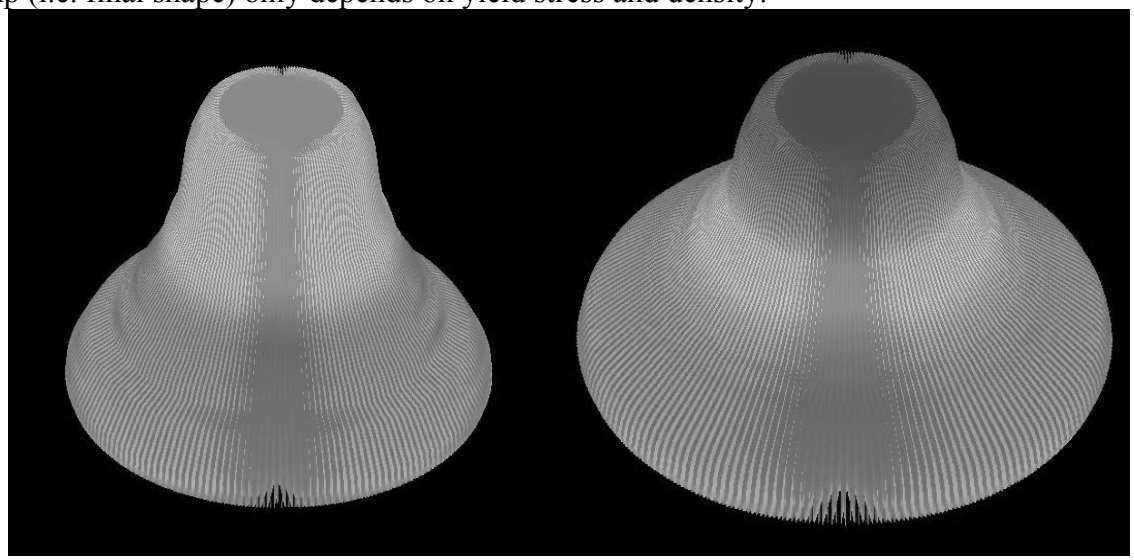

Figure 2. Examples of obtained shapes for the ASTM Abrams cone (left) yield stress = 2600 Pa (right) yield stress = 2000 Pa. Density = 2500 kg/m^3 for both simulations.

3.2. Simulation of concrete casting

Mori and Tanigawa demonstrated the applicability of VDEM to simulate the flow of concrete in a reinforced beam section and the filling of a reinforced wall (2 m high, 3 m long); and Kitaoji *et al.* [9] confirmed the applicability of 2D VDEM to simulate the flow of fresh concrete cast into an unreinforced wall (1m high, 2m long). Kitaoji *et al.* approximated the slipping behavior on the wall surface near the inlet by thin layer elements.

Numerical simulations were also applied to an industrial casting of a very high strength concrete pre-cambered composite beam by Roussel *et al.* [10]. The results of the simulations carried out for various values of the rheological parameters (Bingham model) helped to determine the value of minimum fluidity needed to cast the element. The mix proportioning of the concrete was done keeping in mind this minimum value and the numerical predictions were finally compared with the experimental observations carried out during two trail castings and the real casting of the two 13 m beams (see Fig. 3).

Although the assumptions needed to carry out the simulations may be over-simplistic (the rebars and possible thixotropy were not taken into account and only 2D simulations were carried out), a satisfactory agreement was found between the predicted and actual global flow. However, two neglected phenomena locally perturbed the casting: the thixotropic behaviour of the SCC that induced an increase of the apparent yield stress at low casting speed or when the concrete was at rest and the interaction between the largest aggregates and the steel bars that induced some blocking. The fact that only 2D-simulations were carried out was acceptable in the frame of this work since the shape of the element to be cast was suitable for such a simplifying assumption. However, this is in general not the case and the computational time could strongly increase.

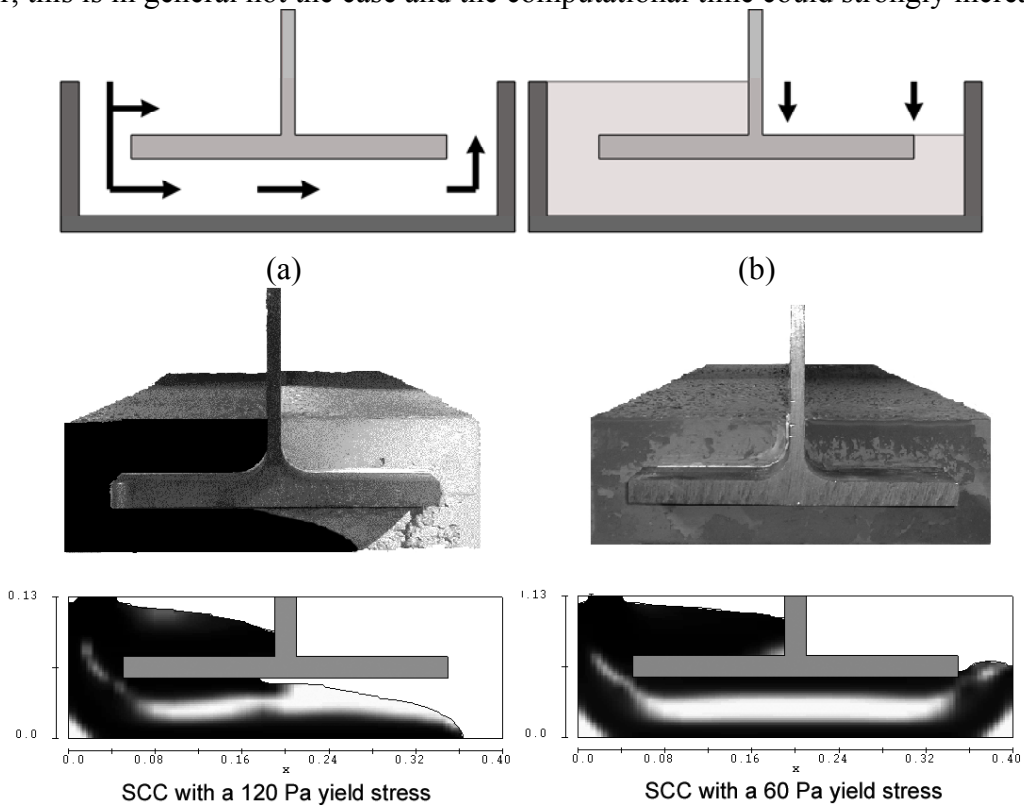

Figure 3. (a) Schematic showing the first part of the casting of a concrete pre-cambered composite beam **(b) Schematic showing the second part of the casting. Comparison between experiments and numerical simulations for two SCC with yield stresses equal to respectively 60 and 120 Pa. Black shading emphasizes the casting defect on the left picture.**

Although most of the above applications deal with yield stress fluids, single fluid methods provide a large choice of behavior laws allowing the simulation of complex phenomena such as

thixotropy. Recently, Roussel [11] proposed a model derived from the Bingham model and able to describe the thixotropy of fresh concrete with only two additional parameters.

$$\tau = (1+\lambda)\tau_0 + \mu_p \dot{\gamma} \quad (1)$$

$$\frac{\partial \lambda}{\partial t} = \frac{A_{thix}}{\tau_0} - \alpha \lambda \dot{\gamma} \quad (2)$$

where A_{thix} is the re-structuration rate of the SCC at rest (Pa/s) and α is a destructuration parameter. λ is the structuration state of the concrete that evolves through the flow history. This model was used to study the two following phenomena:

During placing, the fresh SCC behaves as a fluid but, if cast slowly enough or if at rest, it flocculates and builds up an internal structure and has the ability to withstand the load from concrete cast above it without increasing the lateral stress against the formwork. Numerical simulations were carried out by Ovarlez and Roussel [12] using the above model in order to estimate the extent of the zone where SCC is at rest in the formwork according to the formwork geometry and to the casting rate (see Fig. 4).

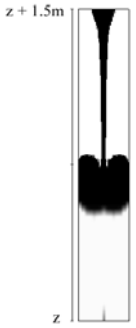

Figure 4. Sheared and resting zone when SCC is cast from the top of the formwork. In the black zone, the shear rate is greater than $0.1s^{-1}$ (from REF).

During placing, a layer of SCC often has a short time to rest and flocculate before a second layer of concrete is cast above it. If the fine particles flocculates too much and the apparent yield stress of the concrete increases above a critical value, the two layers do not combine at all and a weak interface is formed, as vibrating is prohibited in the case of SCC, . Loss of resistance of more than 40% has been reported [13]. Numerical simulations were carried out by Roussel [11] to predict the occurrence of this phenomenon (see Fig. 5).

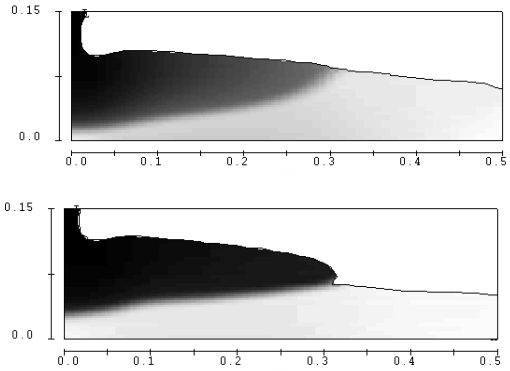

Figure 5. Numerical simulations of the multi-layer casting phenomenon using the model proposed by Roussel [REF] with τ_0 = 50 Pa, μ_p = 50Pa.s, A_{thix} = 0.5 Pa/s, α = 0.005. (a) For a 5 min. resting time, the two layers mix perfectly (b) for a 20 minutes resting time, the two layers do not mix at all.

3.3. Pros and Cons of single fluid simulations

PROS
It is the **fastest way** to simulate the casting of concrete.
The **large choice of behavior law** allows the modeling of many phenomena such as thixotropy.

CONS
Particle blocking and segregation can not be predicted.

4. SIMULATION OF DISCRETE PARTICLE FLOW

An analogy is often made between the flow of liquids and that of granular media, even though the physical properties of the two are quite different. Concrete by nature is dominated by its fluid-like behavior or by its granular media like behavior according to its workability. In the case of SCC for example, the amount of coarse particles in the mixture is low and this modern concrete behaves as a fluid suspension whereas, in the case of ordinary with greater amount of coarse particles, the behavior is dominated by its granular nature.
It is thus relevant to examine the efficiency of distinct element methods used to simulate dry granular media flows to model flow of concrete. Simulation of discrete particle flow has been used for the computational modeling of the movement and interaction of aggregates in concrete and examples are given below.

Standard Discrete (or Distinct) Element Method (DEM)

In the Standard Discrete (or Distinct) Element Method (DEM) method, each calculation cycle includes two steps: the determination of the contact forces between solid particles and the calculation of the motion of each particle determined by the application of Newton's second law. For most applications, each contact point between particles is replaced by a set of normal and shear springs, normal and shear dashpots, normal and shear no tension-joints and shear slider (see Fig. 2). As a complement to laboratory experiments, discrete numerical simulation applied to granular materials gives access to the microstructure at the scale of the grains and contacts, and improves our understanding of the microscopic origin of macroscopic mechanical behavior. The various discrete simulation methods that can be found in the mechanics literature [14], considered as numerical experimentation tools, are presented in relation with mechanical models of inter-granular contacts.

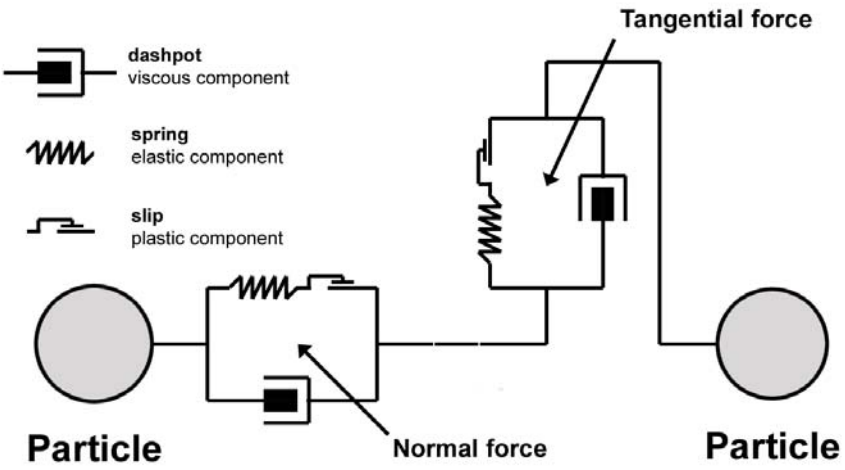

Figure 6. A standard contact law between two particles in DEM.

Based on work by Chu et al. [15], a 3D DEM using a 3D particle flow code program, PCD3D, was applied in a preliminary study by Noor and Uomoto [16] in order to simulate the flow of SCC during various standard tests: slump flow test, L-box and V-funnel. DEM was selected as opposed to a continuum approach and was observed by the authors to reproduce the qualitative behavior in fresh concrete. As a compromise between modeling of aggregate movement and limitation of the computational time, the material was divided into mortar and coarse aggregates larger than 7.5 mm. The method proposed by Noor and Uomoto was also adopted by Petersson and Hakami [17] and Petersson [18] in order to simulate SCC flow during L-box and slump flow test, and J-ring and L-box tests, respectively. They found 3D and, depending on the type of problem, 2D simulations to be applicable.

Dissipative Particle Dynamics (DPD) method

Recently, a new computational method called Dissipative Particle Dynamics (DPD) has been introduced which has several advantages over traditional computational dynamics methods. DPD resembles Molecular Dynamics (MD) in that the particles move according to Newton's laws, but, in DPD, the inter-particle interactions are chosen to allow for much larger time steps. This allows for the study of physical behavior on time scales many orders of magnitude greater than possible with MD. The original DPD algorithm used an Euler algorithm for updating the positions of the free “particles” (which represent "lumps" of fluids), and a leap frog algorithm for updating the positions of the solid inclusions. The mesoscopic particles, which are not to be confused with particles in suspensions, which are called rigid bodies, are subjected to pair-wise forces which comprise: Soft repulsion, stochastic noise and a dissipation (conceptually similar to a viscous drag). Navier-Stokes’ behavior is obtained with a suitable choice of functional form of these potentials, thus the method captures hydrodynamics. Based on work by Koelman and Hoogerbrugge [19], the mesoscopic models have been extended to cover rigid bodies by freezing a part of the fluid particles (the fluid phase). Examples of applications of DPD to the SCC technology are given in [20-22]. DPD has been used to model the sphere movement and interaction of mono-sized [20] and poly-sized spheres (between 200-500 spheres depending on the solid fraction) [22]. The viscosity of the suspensions was calculated based on the average stresses for a given strain rate. The relative calculated viscosities were observed to correlate well with measured data. Compared to analytical composite models, DPD should in combination with

knowledge on the forces between the colloidal particles in the matrix allow modeling of the actual viscosity of the suspension. Furthermore, DPD allows modeling of the effect of mesoscopic heterogeneities as the effects of a slip layer and particle migration during testing in a rheometer. In [21], the flow between two rebars of mono-sized aggregates of either 1/2 or 1/5 of the free space between the rebars is described (see Fig. 7). Modeling of the flow of mono-sized spheres between sets of rebars demonstrated (tendency to) blocking of spheres with diameter at 1/2 of the free space, where as the smaller spheres were observed to flow unhindered. Further studies of the flow are being undertaken by the authors of [21] and [22] incorporating polysized and irregular shaped bodies to approach the geometry of aggregates in SCC.

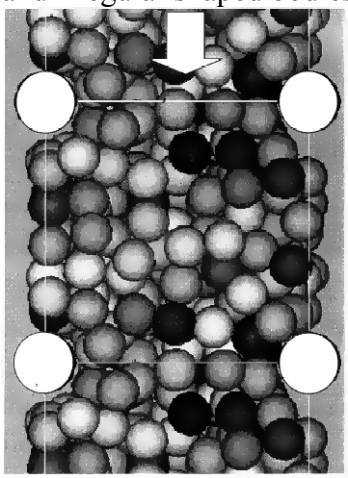

Figure 7. Simulation of flow aggregates between reinforcement bars by Dissipative Particle Dynamics (DPD) [ref]

Pros and cons of simulation of discrete particle flow

PROS

Scale

Limiting cases in which the scale of observation is of the same order as the size of coarsest elements can be dealt with.

Free surface flow

The moving free surface of the flowing material can be modeled in detail.

Study of segregation or mixing

The trajectories of selected species in the mixture can be studied.

CONS

Identification of the parameters

It has to be noted that, although potentially useful, on a physical point of view, the DEM approach, as used in the civil engineering field, may be doubtful as it has no direct physical meaning. It is indeed difficult to define what a direct contact between solid particles that are assumed to represent the mortar phase and solid particles assumed to represent the coarsest grains should be. Moreover, the parameters of the contact law are impossible to measure and can only be fitted in order to get the best comparison between numerical predictions and experiments.

Computation time

Although the DPD approach is very promising in the eternal attempt to understand the correlation between rheological behavior of the material and its mix proportioning, the number of particles

that can be dealt with prevent the use of this technique when prediction of real casting is the objective.

5. SIMULATION OF SUSPENSION FLOW

The concrete is in the following assumed to consist of particles suspended in a fluid.

Viscoplastic Suspension Element Method (VSEM)

Mori and Tanigawa [2] used a so-called Viscoplastic Suspension Element Method (VSEM) to simulate the flow of concrete in various tests. The concrete was divided into mortar and coarse aggregates and the aggregates were approximated as monosize spheres. The rheological properties of the concrete (suspension) were numerically predicted from the rheological properties of the matrix and the coarse aggregate volume fraction. The motion of stiff spheres in a viscoplastic body was simulated by introducing viscoplastic interactions between pairs of particles being closer than a given distance. Kurokawa et al. [23] compared simulated (2D) and measured flow in a L-box and during slump flow test for concretes with yield stresses between 50 and 300 Pa. They found the numerical approach in general to overestimate the initial rate and underestimate the final flow. Mori and Tanigawa tested concretes with minimum yield stress of 100 Pa and claimed the method applicable for the simulation of complex behavior of concrete such as collapse, separation, and mixing.

Finite Element Method with Lagrangian Integration Point (FEMLIP)

Based on the Material Point Method originally developed by Sulsky and Schreyer [24] and belonging to the broad family of Particle-In-Cell method, Moresi *et al.* [25] have recently developed a new hybrid FEM (FEMLIP), where material points and computational points are decoupled. This computational method makes use of a combination of Lagrangian and Eulerian.
The original idea was to keep using FEM well known around the scientific community for its robustness and versatility and modify it to account for infinitely large deformation. The main drawback of Lagrangian FEM for large deformation applications is the element distortion which, once it has reached a critical state, yields inefficiency and inaccuracy of the element-wise integration scheme. Therefore one needs to disconnect material and computational points in order to keep nicely shaped finite elements even for extremely large material deformation. The retained approach uses an Eulerian finite element grid (fixed) as a computational set of points and a set of Lagrangian particles embedded in the mesh which are used as integration points for any given configuration (Fig. 8(b)). Material properties are initially set on particles (Fig. 8(a)). Once nodal unknowns (velocity and pressure for incompressible materials) are computed for any configuration using element-wise integration scheme over particles, the velocity field is interpolated to the particles. In contrary of Gaussian points in classical FEM, shape functions need to be recalculated for each particle configuration as their position are changing through time within elements. Particles positions are updated according to their velocity (Fig. 8(c)). All time dependent properties stored on particles move with the particles and must be rotated if they are tensor variables (e.g. elastic stress tensor). At the end of the computational step the grid contains no information at all (it is a pure computational grid) and one can choose a totally different mesh for the new particle configuration. The grid is usually kept fixed except in the case of moving boundary conditions.

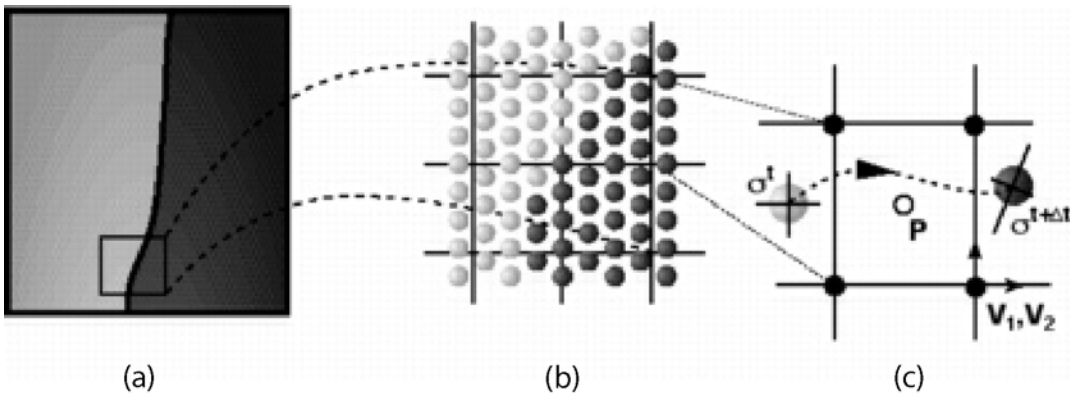

Figure 8. Schematic representation of FEMLIP (a) Interface are set by applying different behavior to material in space (b) A set of Lagrangian particles are used as integration points and to track material properties (c) Particles are moved according to nodal velocities and history dependant variables are stored on particles.

The capacity of FEMLIP to simulate very large deformation processes with interfaces and free surfaces has been demonstrated for a wide range of material properties in several types of applications including concrete flow in forms. When modeling concrete as a heterogeneous material made of mortar and aggregate (see Fig. 9(b)), the scale should be smaller than the form scale since aggregates must be discretised by several finite elements in order to be properly modeled as rigid compared to mortar. Dufour and Pijaudier-Cabot [26] applied this method on two different types of concrete (self-compacting and ordinary concrete). They calibrated the two Bingham's parameters on experimental results from a simple slump test with flow time measurement.

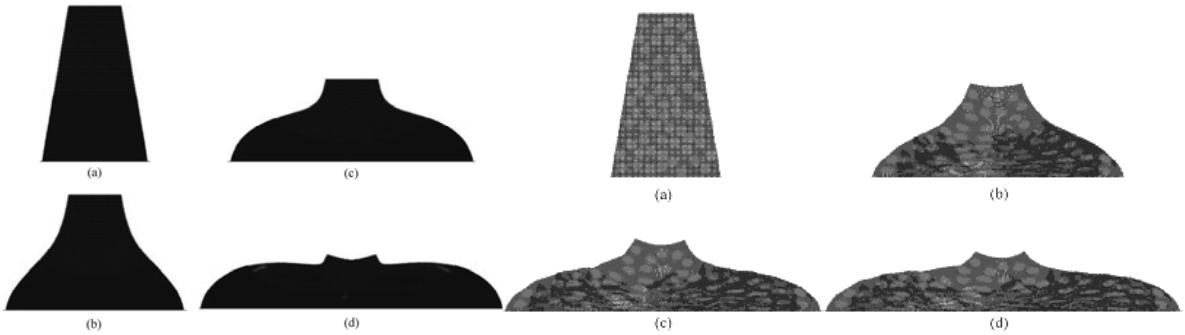

Figure 9. (a) Numerical simulation of the slump flow test by a homogeneous approach (b) Numerical simulation of the slump flow test by a heterogeneous approach.

PROS

Reasonable computational time

- The computational time remains reasonable.

Wide range of constitutive laws

- Any rheological law (Newton, Bingham, Herschell Bulkley, etc …) for mortar.
- Anisotropic viscous model may even be used to represent concrete flow reinforced with fibers.

CONS

- The need to compute all the space where the material of interest has been, is and will move to. However Levelset method could be used to explicitly define the interface between the material of interest and the air that will not have to be modeled any more. The integration domain could be reduced to one side of the levelset (e.g. the half-space of positive isovalues) even when the

levelset run through a finite element as it is in crack propagation problem. Thus it will efficiently reduce the computational cost in time and memory. The key point of such approach is to define the velocity field along the levelset in order to propagate the interface properly.

6. SUMMARY AND CONCLUSIONS

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